The University of Texas at Austin
energy institute

The Full Cost of Electricity (FCe-)

Market-calibrated Forecasts for Natural Gas Prices

PART OF A SERIES OF WHITE PAPERS

TEXAS
The University of Texas at Austin

THE FULL COST OF ELECTRICITY is an interdisciplinary initiative of the Energy Institute of the University of Texas to identify and quantify the full-system cost of electric power generation and delivery – from the power plant to the wall socket. The purpose is to inform public policy discourse with comprehensive, rigorous and impartial analysis.

The generation of electric power and the infrastructure that delivers it is in the midst of dramatic and rapid change. Since 2000, declining renewable energy costs, stringent emissions standards, low-priced natural gas (post-2008), competitive electricity markets, and a host of technological innovations promise to forever change the landscape of an industry that has remained static for decades. Heightened awareness of newfound options available to consumers has injected yet another element to the policy debate surrounding these transformative changes, moving it beyond utility boardrooms and legislative hearing rooms to everyday living rooms.

The Full Cost of Electricity (FCe-) study employs a holistic approach to thoroughly examine the key factors affecting the *total direct and indirect costs* of generating and delivering electricity. As an interdisciplinary project, the FCe- synthesizes the expert analysis and different perspectives of faculty across the UT Austin campus, from engineering, economics, law, and policy. In addition to producing authoritative white papers that provide comprehensive assessment and analysis of various electric power system options, the study team developed online calculators that allow policymakers and other stakeholders, including the public, to estimate the cost implications of potential policy actions. A framework of the research initiative, and a list of research participants and project sponsors are also available on the Energy Institute website: energy.utexas.edu

All authors abide by the disclosure policies of the University of Texas at Austin. The University of Texas at Austin is committed to transparency and disclosure of all potential conflicts of interest. All UT investigators involved with this research have filed their required financial disclosure forms with the university. Through this process the university has determined that there are neither conflicts of interest nor the appearance of such conflicts.

This paper is one in a series of Full Cost of Electricity white papers that examine particular aspects of the electricity system.

Other white papers produced through the study can be accessed at the University of Texas Energy Institute website: **energy.utexas.edu**

The University of Texas at Austin
energy institute

Market-calibrated Forecasts for Natural Gas Prices

Warren J. Hahn, *The University of Texas at Austin*

James A. DiLellio, *Pepperdine University*

James S. Dyer *The University of Texas at Austin*

The University of Texas at Austin

Hahn, Warren J., DiLellio A., James, Dryer S., James, "Market-calibrated Forecasts for Natural Gas Prices," White Paper UTEI/2016-07-1, 2016, available at http://energy.utexas.edu/the-full-cost-of-electricity-fce/.

ABSTRACT:

Stochastic process models of commodity prices are important inputs in energy investment evaluation and planning problems. In this work, we focus on modeling and forecasting the long-term price level, since it is the dominant factor in many such applications. We apply a Kalman filtering method with maximum likelihood approach to estimate the model parameters for the two-factor Schwartz and Smith (2000) process, which decomposes the spot price into unobservable factors for forecasting the long-term equilibrium level and short-term deviation. The method also accommodates aspects of both a geometric Brownian motion process and a mean-reverting process. Historical natural gas futures data from 1996 to present were analyzed to determine the model parameters and we quantified the changes in both the drift rate and volatility that have resulted from developments in the natural gas markets since significant volumes of shale gas began to be produced. The parameterized model is then used to develop price forecasts with uncertainty bounds. The risk-neutral version of the stochastic price model is typically used theory and in academic work; however, risk-adjusted models of the expected spot price are often used in practice. These two approaches are connected by risk premia which are unfortunately often difficult to estimate. We use an asset pricing model approach to obtain improved estimates of the risk premia to facilitate development of both risk-neutral and expected spot price forecasts.

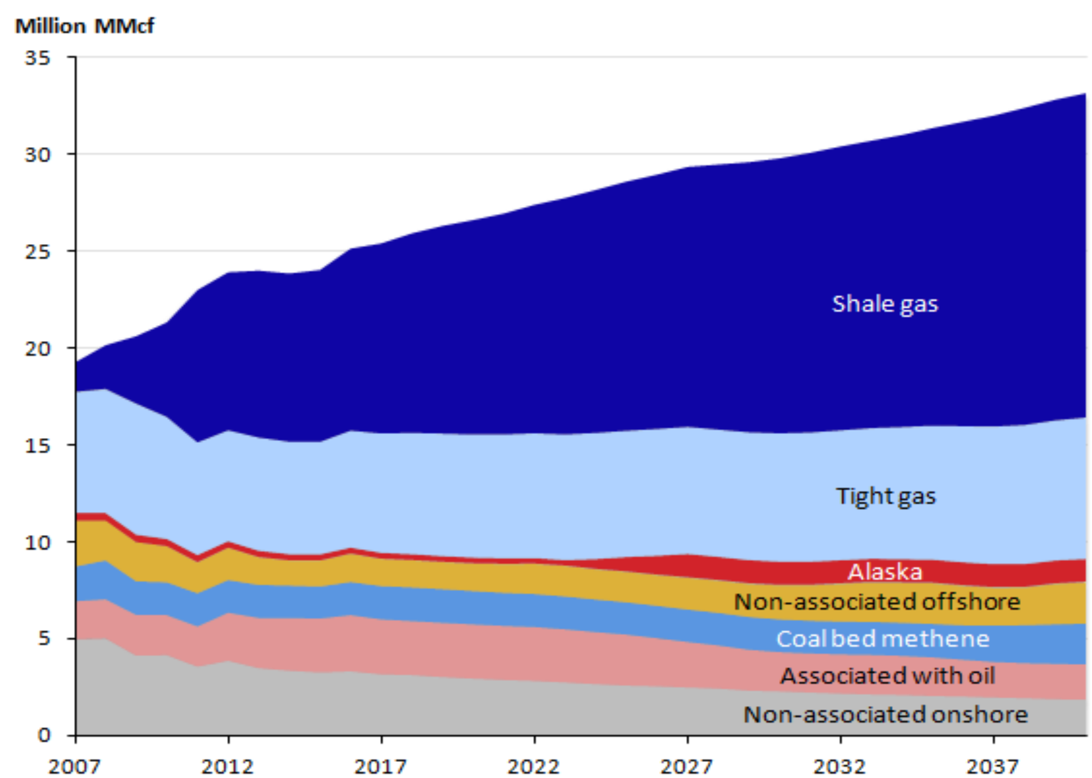
1 | INTRODUCTION

The renewed and enhanced application of horizontal drilling and hydraulic fracturing technologies to shale reservoirs has dramatically changed the US domestic natural gas production output during the past decade. Prior to 2008, it was expected that the US would need to import large volumes of liquefied natural gas (LNG) to make up for an anticipated shortfall in domestic production. The International Energy Agency (IEA) now projects that the United States will be a net exporter of natural gas as soon as 2020. The gas price environment, through its impact on investment economics, will undoubtedly be a key factor in determining whether such projections are accurate or not. Figures 1 and 2 show the historical and future projected impact of shale gas production and the resulting realized price, respectively. The fundamental economic relationship between supply and demand seems to hold, with the increase in production coinciding with a decrease in the price index.

There are several approaches to developing long-term forecasts for commodity prices, including many types of econometric models, equilibrium models, and expert survey forecasts. In this paper, we use an approach that is based upon calibrating some of the commonly-used stochastic process models with data from the commodities markets. Schwartz (1997), Schwartz and Smith (2000), Manoliu and Tompaidis (2002), and others describe how the parameters for these types of process models can be obtained with the Kalman filter and maximum likelihood estimation, and evaluate

FIGURE 1:

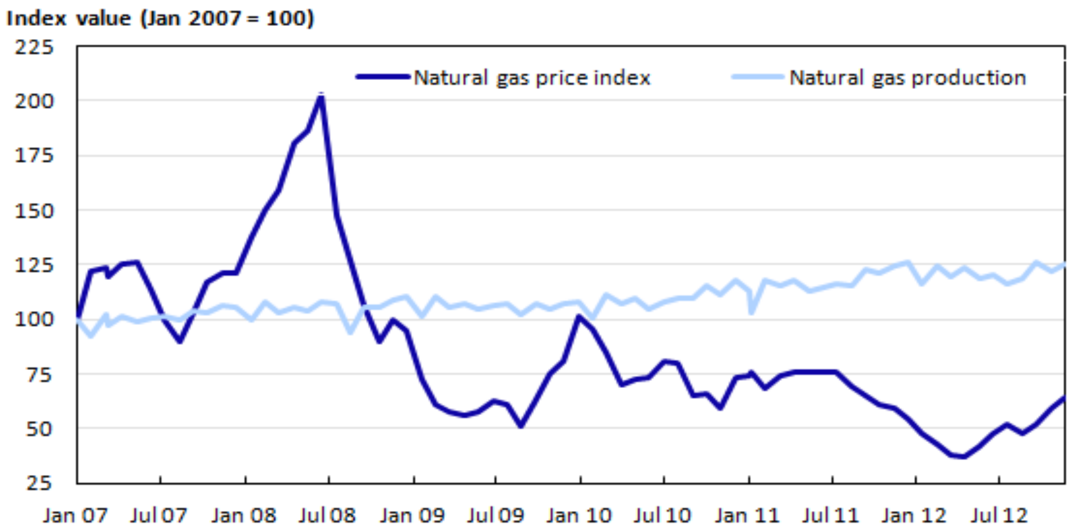
US Domestic Natural Gas Production 2007-2040



Note: Data for 2012–2040 are projections.
Source: Energy Information Administration.

FIGURE 2:

Natural Gas Price and Production Indices 2007-2012



Sources: U.S. Bureau of Labor Statistics and the Energy Information Administration.

the performance of these models for capturing the dynamics of futures prices [1-3]. We use this approach to calibrate the Schwartz and Smith (2000) two-factor stochastic process and then use that model to generate forecasts of natural gas spot prices. The relationship between futures and spot prices in this approach is established within the context of a risk-neutral valuation framework, where futures prices are equal to the expected future spot price under a risk-neutral stochastic process [4].

2 | TWO-FACTOR STOCHASTIC PROCESS MODEL

For the Schwartz and Smith (2000) two factor model, the natural log of the natural gas price at any point in time S_t is expressed as the sum of short-term deviations χ_t and a long-term equilibrium level ξ_t, such that $\ln(S_t) = \chi_t + \xi_t$. Assuming the short-term deviations follow a mean-reverting process and the long-term equilibrium follows geometric Brownian motion, the two factor process can be summarized as:

(1)

$$d\chi_t = \kappa(0 - \chi_t)dt + \sigma_\chi \, dz_\chi$$

(2)

$$d\xi_t = \mu_\xi \, dt + \sigma_\xi \, dz_\xi$$

where κ is the reversion rate of the deviation to an mean value of zero, σ_X is the volatility of the short term deviation, μ_ξ is the drift rate of the long term equilibrium level, and σ_ξ is the volatility of the long term equilibrium. The random increments of the two standard Brownian motion processes, dz_X and dz_ξ, are correlated according to the relationship $dz_\chi dz_\xi = \rho_{\chi\xi} dt$.

The calibration of the two factor process utilizes futures prices, and because futures prices are equal to the expected future spot price under a risk-neutral stochastic process (Duffie, 1992), it is necessary to develop a risk neutral version of the two factor process. This version can also then be used to value investments that derive their value from the underlying commodity (i.e., natural gas in our case), including real options, without the requirement of estimating a risk-adjusted discount rate. The modifications to Equations (1) and (2) for a risk neutral process involve relatively straightforward adjustments to the drift rates of the processes for χ_t and ξ_t using risk premiums λ_χ and λ_ξ, respectively:

(3)

$$d\chi_t = (0 - \kappa \, \chi_t - \lambda_\chi)dt + \sigma_\chi \, dz^*_\chi$$

(4)

$$d\xi_t = \mu^*_\xi \, dt + \sigma_\xi \, dz^*_\xi$$

where the parameters are defined the same as in Equations (1) and (2), except μ^*_ξ is the risk neutral drift rate of the long term equilibrium level, calculated as $\mu^*_\xi = \mu_\xi - \lambda_\xi$ and the random increments of the two processes, dz^*_χ and dz^*_ξ are increments of the risk neutral process.

The expectation and variance of the risk neutral process, as shown in Schwartz and Smith (2000) are:

(5)

$$E[\ln(S_t)] = e^{-\kappa t} \chi_0 + \xi_0 - (1 - e^{-\kappa t}) \frac{\lambda_\chi}{\kappa} + \mu^*_\xi t$$

(6)

$$Var[\ln(S_t)] = (1 - e^{-2\kappa t}) \frac{\sigma^2_\chi}{2\kappa} + \sigma^2_\xi t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi \sigma_\xi}{\kappa}$$

Here, S_t is lognormally distributed, so that $\ln(S_t/S_0)$, which is the return prices from period 0 to period t, is normally distributed. Under risk-neutral valuation, the futures prices will equal the expected spot prices [5]. Therefore the expectation and variance, in (5) and (6), can be used to derive the following expression for the futures prices:

$$\text{Ln}(F_{T,0}) = e^{-\kappa t} \chi_0 + \xi_0 + A(T)$$

where $F_{T,0}$ is the current (time 0) market price for a futures contract with maturity at time T, and

$$A(T) = \mu^*_\xi T - (1 - e^{-\kappa t}) \frac{\lambda_\chi}{\kappa} + \frac{1}{2}\left[(1 - e^{-2\kappa T}) \frac{\sigma^2_\chi}{2\kappa} + \sigma^2_\xi t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi \sigma_\xi}{\kappa}\right]$$

3 TWO-FACTOR MODEL PARAMETERIZATION AND PRICE FORECAST

There are seven parameters required to fully specify the model in its basic and risk neutral versions. We estimate the values of these parameters using a Kalman filtering with maximum likelihood estimation. The Kalman filter is a recursive procedure for optimally estimating unobserved state variables based on observations that depend on these state variables [6]. In this case, the Kalman filter can be applied to estimate the unobservable state variables χ_t and ξ_t, making it possible to calculate the likelihood of a set of observations given a particular set of parameter values. By varying the parameter values and re-running the Kalman filter, the value of the parameters that maximize the likelihood function can be identified. A detailed description of this technique can be found in Harvey (1989) [7].

For the Kalman filter, the stochastic process must be represented in a state space formulation. This representation consists of a transition equation to describe the evolution of the state variables over time and a measurement equation to relate the state variables to the observable data. Schwartz and Smith (2000) specify the *transition equation* for the two factor model as:

$$x_t = c + Gx_{t-1} + \omega_t, \quad t = 1, \dots, n_T$$

Where n_T is the number of time periods,

$x_t = \begin{bmatrix} \chi_t \\ \xi_t \end{bmatrix}$ is a 2x1 vector of state variables;

$c = \begin{bmatrix} 0 \\ \mu_\xi \Delta t \end{bmatrix}$ is a 2x1 vector and is the length of time steps;

$G = \begin{bmatrix} e^{-\kappa t} & 0 \\ 0 & 1 \end{bmatrix}$ is a 2x2 transition matrix;

ω_t is a 2x1 vector of serially uncorrelated normally-distributed disturbances, with

$E[\omega_t] = 0$, and

$$Var[\omega_t] = Cov[\chi_{\Delta t}, \xi_{\Delta t}] = \begin{bmatrix} (1 - e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} & (1 - e^{-2\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \\ (1 - e^{-2\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} & \sigma_\xi^2 t \end{bmatrix}.$$

The corresponding *measurement equation* is:

$$y_t = d_t + F_t' x_t + v_t, \quad t = 1, \dots, n_T,$$

where $y_t = \begin{bmatrix} \ln(F_{T,1}) \\ \vdots \\ \ln(F_{T,n}) \end{bmatrix}$ is a n x 1 vector of observed (log) futures prices for the n maturities $T_1, T_2, \dots, T_n$,

$d_t = \begin{bmatrix} A(T_1) \\ \vdots \\ A(T_n) \end{bmatrix}$ is a n x 1 vector, $F_t' = \begin{bmatrix} e^{-\kappa T_1} & 1 \\ \vdots & \vdots \\ e^{-\kappa T_n} & 1 \end{bmatrix}$ is a n x 2 matrix, and v_t is a n x 1 vector of serially uncorrelated normally-distributed disturbances (measurement errors) with

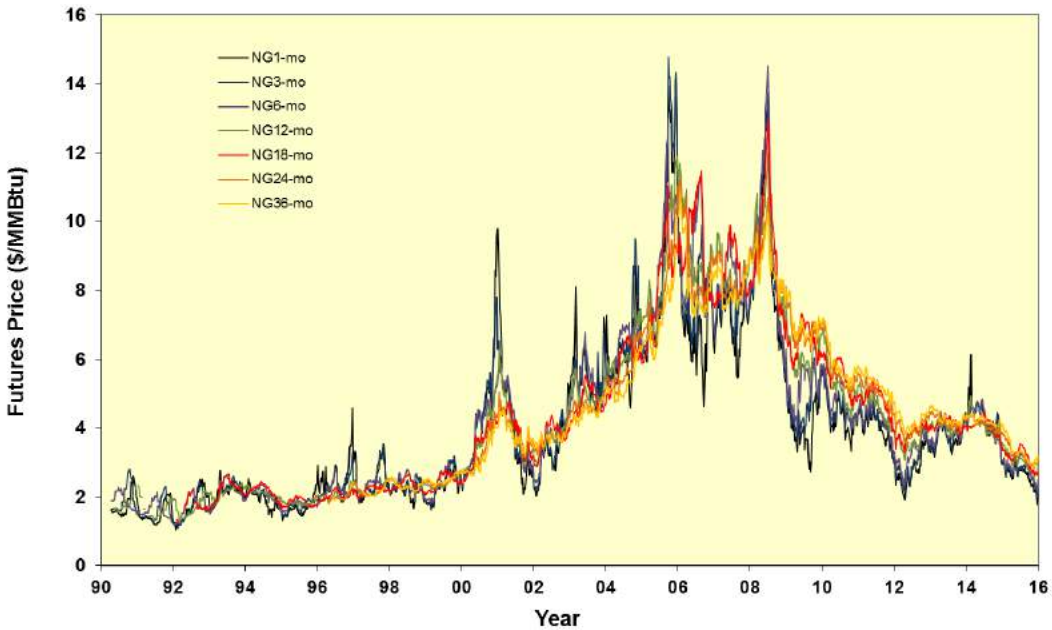
$E[v_t] = 0$ and $Cov[v_t] = V$.

With a state space formulation and a set of historically observed futures prices for different maturities, the Kalman filter runs recursively beginning with a prior distribution of the initial values of the state variables (χ_0, ξ_0). In addition, the terms in the covariance matrix (V) for the measurement errors for each of the futures contract maturities in the data must also be estimated. The measurement errors can be simplified by making the common assumption that they are uncorrelated with each other, so that V is a diagonal matrix as in Schwartz (1997) and Schwartz and Smith (2000). The objective is to maximize the log-likelihood function for a joint normal distribution.

Our sample of futures data consisted of 969 weekly observations of futures prices at maturities of 1, 3, 6, 12, 18, 24 and 36 months, as shown in Figure 3. The time period for these observations is from the week of June 6, 1997, the date from which all seven contracts above were first continuously traded, to the week of January 1, 2016. We also worked with a subset of this data, which included

FIGURE 3:

Natural gas futures price data



366 weekly observations, from the week of January 2, 2009 through the same end date as the full data set. This subset was selected to correspond with the estimated date of the effective beginning of the shale gas era, because it coincides with the approximate date when natural gas produced by hydraulic fracturing started to significantly influence market prices.

The data from both time horizons (from June 1997 and from January 2009) were analyzed using the Kalman filter to maximize the likelihood function and produce the parameter estimates reported in Table 1. To characterize the uncertainty in the parameter estimates, the standard errors are also shown in Table 1.

Several observations regarding the risk premia in the two-factor model can be made from reviewing Table 1. First, the long-term equilibrium drift rate

TABLE 1:

Parameter estimates and standard errors

	1997-2016 Data Set		2009-2016 Data Set	
	Parameter Estimate	SE(Est.)	Parameter Estimate	SE(Est.)
μ_ξ	-0.0465	0.0519	-0.2060	0.0605
κ	0.9451	0.0197	1.3998	0.0270
λ_χ	0.4794	0.0480	-0.7452	0.0489
σ_χ	0.4961	0.0192	0.3770	0.0201
σ_ξ	0.2223	0.0058	0.1628	0.0062
$\rho_{\xi\chi}$	-0.2534	0.0480	0.3320	0.0672
μ_ξ^*	-0.0380	0.0019	0.0159	0.0014
$\lambda_\chi = \mu_\xi - \mu_\xi^*$	-0.0085		-0.2219	

TABLE 2:

Beta coefficients and asset pricing model expected returns

Contract Maturity (days)	1997-2016 Data Set			2009-2016 Data Set		
	β	t-statistic	$E[R_T]$	β	t-statistic	$E[R_T]$
30	0.1277	1.11	0.51%	0.4165	2.32	1.67%
90	0.1167	1.21	0.47%	0.3696	2.49	1.48%
180	0.1564	2.14	0.63%	0.2841	2.26	1.14%
360	0.1272	2.26	0.51%	0.2769	3.07	1.11%
540	0.1397	2.79	0.56%	0.2149	2.85	0.86%
720	0.1225	2.61	0.49%	0.2207	3.39	0.88%
1080	0.1241	2.72	0.50%	0.2234	3.74	0.89%

term μ_ξ is negative for both data sets, especially for the shale gas era data set. As noted by Schwartz and Smith (2000) and others, a negative estimate for this drift rate may not be realistic, given that even a small inflation rate over the forecast period should generate a slightly upward-trending forecast. In addition, the standard errors of these estimates are relatively large, indicating that these terms were not estimated very precisely. This limited observability in the drift rate of the long term equilibrium level has also been documented by others, including Schwartz and Smith (2000). Although the risk neutral long-term drift rate term μ^*_ξ is more accurately estimated in both data sets, the calculated values for λ_ξ, shown in the bottom row of Table 1, cannot be precisely estimated due to the uncertainty in μ_ξ.

The short term risk premium λ_χ estimates from both data sets also had relatively high standard errors, but the larger issue for this parameter was the variability of the estimates from the two data sets, and apparent sensitivity to the data set time horizon. Other researchers have noted similar issues with estimating this risk premium parameter (see for example [8]). The implication of these issues is that, while it would be possible to parameterize a risk neutral price model, which does not depend on the estimated risk premia[1], it would not be possible to specify a reliable model of the expected spot price without better estimates of λ_χ and λ_ξ.

To improve the estimates λ_χ and λ_ξ, Cortazar et al. (2015) suggest the use of one or more of three different approaches to exogenously estimate these risk premia. The three approaches are 1) using capital asset pricing model (CAPM)-like asset pricing model to estimate the risk premia, 2) setting the risk premia to zero and 3) using expert opinion of the values of the risk premia. In this work, we used the asset pricing model approach, as it is most likely to align with our goal of obtaining market valuations.

The asset pricing model approach is analogous to estimation of β for the capital asset pricing model (CAPM) risk premium for a stock. The variable β characterizes if a given asset is more or less variable than the market overall. If $\beta < 1$ the asset price is less volatile, and if $\beta > 1$ the asset price is more

1 It may appear that the risk premia, λ_χ and λ_ξ, are required to parameterize the risk neutral version of the two factor model because Equation 8a of Schwartz and Smith (2000) includes both the risk neutral drift rate of the equilibrium level, which is connected to the true drift rate of the equilibrium level by the long-term risk premium ($\mu^*_\xi = \mu_\xi - \lambda_\xi$), as well as the risk premium for the short term deviation, λ_χ. However, neither of the risk premia has any effect in the risk-neutral model. The risk-neutral drift rate for the long term equilibrium level μ^*_ξ is directly estimated by the Kalman Filter, therefore λ_ξ does not directly or indirectly factor into Equation 8a. While λ_χ does appear in Equation 8a, any changes to it must be accompanied by adjustments to the values of the two factors (χ_0 and ξ_0) and there is no net effect on the risk-neutral distribution of prices. This relationship is discussed in detail in Section 6.1, p. 906 of Schwartz and Smith (2000).

TABLE 3:

Risk premia estimates from the asset pricing model approach

	1997-2016 Data Set			2009-2016 Data Set		
Contract Maturity (days)	APM $E[R_T]$	Eq. (7) Calculated $E[R_T]$	SSE	APM $E[R_T]$	Eq. (7) Calculated $E[R_T]$	SSE
30	0.0051	0.0053	2.72E-08	0.0167	0.0162	1.93E-07
90	0.0047	0.0053	3.48E-07	0.0148	0.0145	5.59E-08
180	0.0063	0.0052	1.03E-06	0.0114	0.0127	1.71E-06
360	0.0051	0.0052	1.64E-08	0.0111	0.0104	4.30E-07
540	0.0056	0.0052	1.48E-07	0.0086	0.0093	4.98E-07
720	0.0049	0.0052	8.72E-08	0.0088	0.0087	6.45E-09
1080	0.0050	0.0052	4.84E-08	0.0089	0.0083	3.60E-07
		Sum =	1.70E-06		Sum =	3.25E-06
	λ_χ	**0.01%**		λ_χ	**0.41%**	
	λ_ξ	**0.52%**		λ_ξ	**0.82%**	

volatile. In our case, we analyze seven assets which are the seven different maturity futures contracts. The different maturities facilitate estimation of the short and long term risk premia. To calculate the seven beta coefficients, we regressed the weekly returns for each futures contract against a market proxy, the Vanguard 500 Index (VFINX). The beta coefficients were then multiplied by the average market premium since 2000 (4%, from Graham and Harvey, 2012) to produce the expected return for each contract, with the results for both of our data sets shown in Table 2.

The vectors of expected futures contract returns from Table 2 can then be equated with the expected contract returns from the Schwartz and Smith (2000) model, which was shown by Cortazar et al. (2015) to be:

(7)

$$E_0[r_{T},_{\Delta t}] = exp\left(\lambda_\xi \Delta t - e^{-\kappa t}(1 - e^{\kappa \Delta t})\frac{\lambda_\chi}{\kappa}\right) - 1$$

This yields a system of seven equations with three unknowns; κ, λ_χ and λ_ξ. We used the κ estimates from Table 1 for each data set and solved for the risk premia numerically with Excel Solver by minimizing the sum of the squared differences between the expected futures return for each contract in Table 2 and the calculated expected returns from Equation (7). The results of this

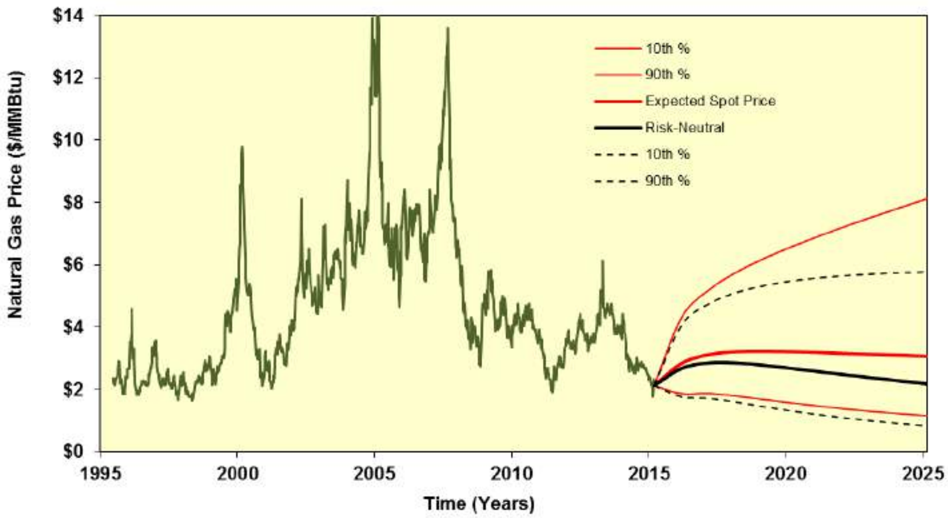
TABLE 4:

Parameter estimates and standard errors for restricted case

	1997-2016 Data Set		2009-2016 Data Set	
	Parameter Estimate	SE(Est.)	Parameter Estimate	SE(Est.)
μ_ξ	-0.0372	n/a	0.0228	n/a
κ	0.9481	0.0199	1.4010	0.0269
λ_χ	0.0001	n/a	0.0041	n/a
σ_χ	0.4556	0.0170	0.4139	0.0226
σ_ξ	0.2318	0.0061	0.1768	0.0075
$\rho_{\xi\chi}$	-0.2957	0.0421	0.0890	0.0639
μ_ξ^*	-0.0424	0.0020	0.0146	0.0017
$\lambda_\chi = \mu_\xi - \mu_\xi^*$	0.0052		0.0082	

FIGURE 4:

Natural gas historical and forecasted prices (1996 – 2016 data set)



analysis are shown below in Table 3, again, for both data sets. Relative to the estimates in Table 1, these estimates show more reasonable values for the risk premium for the long term equilibrium level, and less sensitivity to the data time horizon with the risk premium for the short term deviation.

With the asset pricing model estimates for the risk premia, a restricted case can then be run where only the remaining five parameters are estimated by the Kalman filter, and the two risk premia are entered as deterministic model inputs. Our results for the restricted case for both data set, which are summarized in Table 4, can then be compared against the unrestricted case results in Table 1.

Using the parameter estimates in Table 4, we can now develop forecasts and confidence envelopes for both the risk neutral price and the expected spot price, which are shown graphically in Figures 4 and 5 and numerically in Table 5. For both data sets, we can see the effect of the risk premia through the

FIGURE 5:

Natural gas historical and forecasted prices (2009 – 2016 data set)

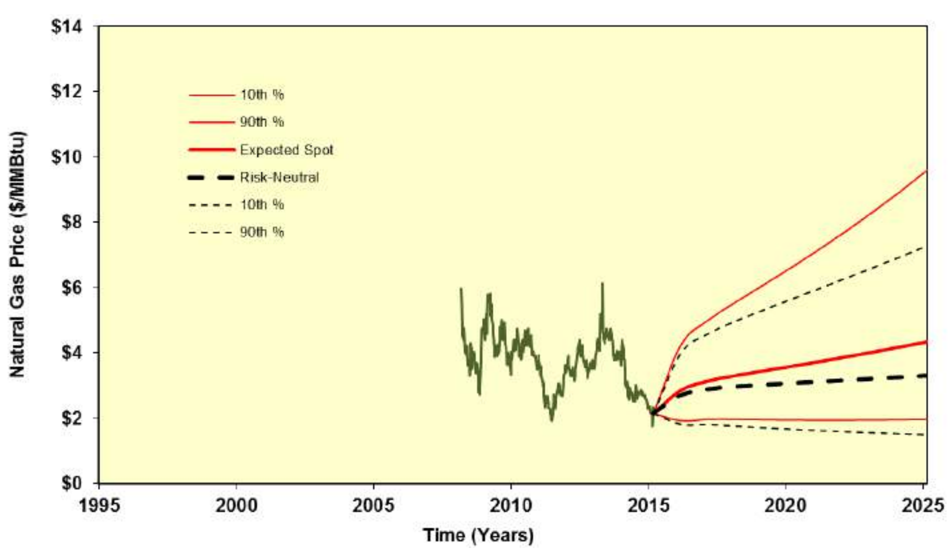


TABLE 5:

Forecasted natural gas spot prices and confidence envelope

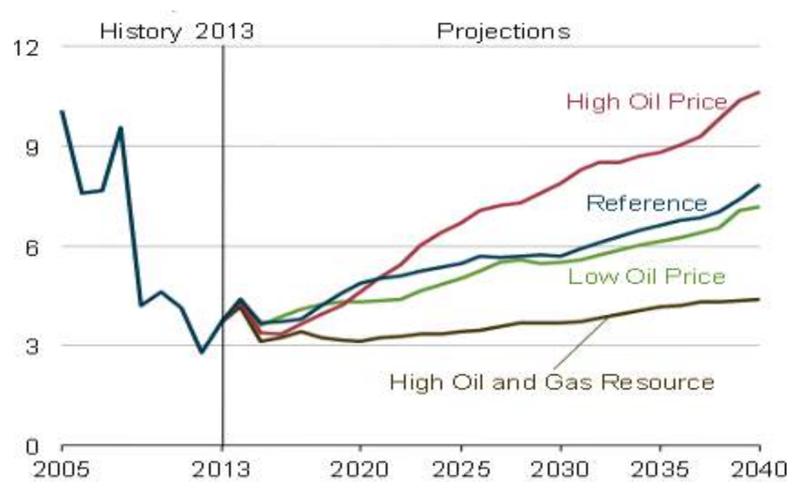
	1996 - 2016 Data Set			2009 - 2016 Data Set		
	Spot Price	Envelope		Spot Price	Envelope	
Year	S_t	10th %	90th %	S_t	10th %	90th %
2016	$2.12			$2.15		
2017	$2.83	1.87	4.30	$2.86	1.92	4.25
2018	$3.12	1.87	5.19	$3.14	1.97	5.01
2019	$3.21	1.79	5.76	$3.31	1.97	5.56
2020	$3.23	1.68	6.20	$3.45	1.96	6.08
2021	$3.21	1.57	6.58	$3.59	1.95	6.61
2022	$3.19	1.47	6.92	$3.73	1.95	7.16
2023	$3.16	1.38	7.24	$3.88	1.95	7.73
2024	$3.12	1.29	7.55	$4.03	1.95	8.33
2025	$3.09	1.22	7.84	$4.19	1.96	8.96
2026	$3.06	1.15	8.13	$4.35	1.97	9.62

slightly lower risk neutral forecasts. In Figure 4, it is apparent that the long term equilibrium level drift in the two factor model is influenced by the by the longer term (i.e., back to circa 2000) historical trend from a period of high, and occasionally very high, prices (circa 2000 to 2008) to a period of low and declining prices (circa 2009 to present). As a result, the expected spot price is forecasted to rise to just above $3.00/million Btu during the next two years and then stay nearly flat at this level out to the end of the forecast horizon. This may be interpreted as a somewhat conservative forecast, although it roughly aligns with the High Oil and Gas Resource scenario projections from the EIA 2015 Energy Outlook (Figure 6).

The forecast in Figure 5, by comparison, is not influenced by the memory of high longer term historical price levels and downward trend. During the period from 2009 through 2014, spot prices oscillated around a mean price just under $4/million Btu, and then prices dropped significantly and rapidly in 2015. As a result, the expected spot price is forecasted to recover to about $3.00/million Btu over the next two years, but then grow at a moderate rate to a price of $4.35/million Btu by the end of the forecast horizon. This forecast is less conservative than the forecast in Figure 4, although it is still below the Reference case projections from the EIA 2015 Energy Outlook in Figure 6 below.

FIGURE 6:

EIA scenarios and projections for Henry Hub natural gas spot prices



Source: EIA Annual Energy Outlook 2015

4 | CONCLUSIONS

In this paper, we used natural gas futures prices in a Kalman filter – maximum likelihood estimation approach to parameterize the Schwartz and Smith (2000) stochastic process model. We find, as have other researchers, that the short and long term risk premia are not well estimated by the Kalman filter approach. We also find that an asset pricing model approach can be used to obtain improved estimates of these parameters, and that the Kalman filter can be used on a restricted basis to estimate the remaining two factor model parameters. The estimates of these remaining parameters and their standard errors are not significantly changed in the restricted case. The improved estimation of the risk premia allows development of reasonable forecasts for both the risk neutral price and the expected spot price.

We applied this approach to parameterize the Schwartz and Smith (2000) model using a large data set of natural gas futures prices, beginning in 1996 when all of our contracts of interest began to be continuously traded, as well as a more abbreviated data set that is intended to represent the shale gas production era, which began to impact volumes and prices in the 2009 timeframe. We found that the choice of data set has some effect on the two factor model parameter estimates and the resulting forecast, with the longer term data set resulting in a slightly lower forecast due to the long term downward trend from the high prices realized in the mid- to latter part of the 2000-2010 decade. With either data set, however, we obtain forecasts that roughly align with the High Oil and Gas Resource and Low Oil Price scenarios from the 2015 EIA Energy Outlook, two outcomes that seem increasingly likely as judged by market sentiment. This market-based forecasting model provides the added benefits of simple updating (as new futures data becomes available) and a statistical basis for uncertainty analysis, through the confidence envelope around the future expected spot prices. ■

REFERENCES:

[1] Schwartz, E., 1997. The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of Finance* **52(3),** 923-973.

[2] Schwartz, E. and Smith, J., 2000. Short-term variations and long-term dynamics in commodity prices. *Management Science* **46(7),** 893-911.

[3] Manoliu, M., Tompaidis, S., 2002. Energy futures prices: term structure models with Kalman filter estimation. *Applied Mathematical Finance* **9(1)**, 21-43.

[4] Duffie, D., 1992. Dynamic asset pricing theory. Princeton University Press, Princeton, NJ.

[5] Black, F., 1976. The pricing of commodity contracts. Journal of Financial Economics **3(1-2)**, 167-179.

[6] Kalman, R., 1960. A new approach to linear filtering and prediction problems. *Transactions of the ASME – Journal of Basic Engineering* **82(1)**, 35-45.

[7] Harvey, A., 1989. Forecasting, structural time series models and the Kalman filter. Cambridge University Press, Cambridge, U.K.

[8] Cortazar, G., Kovacevik, I., and Schwartz, E., 2015. Expected commodity price returns and pricing models. *Energy Economics* **49(May 2015),** 60–71.

The University of Texas at Austin
energy institute